

# The Impact of the Higgs on Einstein's Gravity

M.D. Maia

University of Brasilia, Institute of Physics, Brasilia 70910-900, maia@unb.br

Valdir B. Bezerra

Federal University of Paraiba, Department of Physics, Joao Pessoa, 58051-900, valdir@fisica.ufpb.br

(Dated: March 1, 2017)

The existence of the Higgs particle necessarily implies in the inclusion of Einstein's gravitational field in the standard model of interactions. It also implies in the existence of a new massive, spin-2, weakly interacting field of geometrical nature, acting as a short range carrier of Einstein's gravitation.

The experimental observation of the Higgs particle at the LHC has confirmed that the Higgs mechanism is a natural phenomenon, through which the particles of the standard model of interactions (smi) acquire their masses from the spectrum of eigenvalues of the Casimir mass operator of the Poincaré group. The fact that the masses and orbital spins defined by the Poincaré group appear in particles of that model, consistent with the internal (gauge) symmetries, naturally suggests the existence of some kind of combination between all symmetries of the total Lagrangian. However, such "symmetry mixing" sits at the core of an acute mathematical problem which emerged in the 1960's, after some "no-go" theorems showed the impossibility of an arbitrary combinations between the Poincaré group with the internal symmetries groups. More specifically, it was shown that the particles belonging to the same internal spin multiplet would necessarily have the same mass, in complete disagreement with the observations [1, 2].

It took a considerable time to understand that the problem was located in the somewhat destructive "nilpotent action" of the translational subgroup of the Poincaré group over the spin operators of the electroweak symmetry  $U(1) \times SU(2)$  [3, 4]. Among the proposed solutions, one line of thought suggested a simple replacement of the Poincaré group by some other Lie symmetry, like for example the 10-parameter homogeneous de Sitter groups. Another, more radical proposal suggested the replacement of the whole Lie algebra structure by a graded Lie algebra, in the framework of the super-string program. Such propositions have impacted on the subsequent development of high energy physics and cosmology during the next four or five decades, lasting up to today. Here, following a comment by A. Salam [5], we present a new view of the symmetry mixing problem, based on the Higgs vacuum symmetry. In order to assign masses to all particles of the smi, in accordance with the eigenvalues of the Casimir mass operator of the Poincaré group, the vacuum symmetry must remain an exact symmetry mixed with the Poincaré group. Admittedly, this is not too obvious because the Higgs mechanism requires the breaking of the vacuum symmetry and consequently also of the mixing. We start with the analysis of the Higgs

vacuum symmetry, and its relevance to the solution of the symmetry mixing problem. In the sequence, we explore the fact that the mixing with the Poincaré group also implies in the emergence of particles with higher spins, including the relevant case of the Fierz-Pauli theory of spin-2 fields in the smi. We end with the proposition of a new, massive spin-2 particle of geometric nature, acting as a short range carrier of the gravitational field, complementing the long range Einstein's gravitational interaction.

We begin by tracing an analogy between the "Mexican hat" shape of the Higgs potential with a cassino roulette. The roulette works by the combined action of gravitation with the spin produced by the action of the croupier over the playing ball. The energy of the ball eventually ends as it "naturally falls" into one of the numbered slots at the bottom of the roulette, producing a winning number. In our analogy, the playing ball represents a particle of the standard model and the numbered slots at the bottom of the roulette corresponds to Higgs vacuum represented by a circumference at the bottom of the hat, whose symmetry group is  $SO(2)$ . A difference is that while the slots in the roulette are labeled by the integers, the bottom circle of the Mexican hat is a continuous manifold parametrized by an angle, assuming specific real values in the interval  $[0, \infty)$ . When a particle falls into the vacuum, it "wins a mass" so to speak, not any mass, but only a discrete, positive, isolated real mass values which correspond to one of the eigenvalues of the Casimir mass operator of the Poincaré group [27]. In other words, the measurement of one particle mass in its vacuum state is an "observational condition" of the Higgs theory, which in our analogy corresponds to stopping the roulette, so that every player can read and confirm who is the winner, does not end the game. The roulette will spin again, so that all other particles also may have the chance of winning a mass. The spontaneous breaking of the vacuum symmetry will does not eliminate that symmetry. Consequently, the Higgs mechanism requires that the vacuum symmetry is exact, braking only at the moment of assigning the mass to any given particle.

in the existence of a 1:1 correspondence between the spectrum of mass eigenvalues of the Poincaré group and

a set of isolated points in the Higgs vacuum, identified by specific values of the angle parameter of the  $SO(2)$  vacuum symmetry. Since the latter group is isomorphic to the gauge group  $U(1)$ , the result is an indirect symmetry mixing, in which the Poincaré translations are replaced by the rotations of the  $SO(2)$  vacuum symmetry, without harming the internal spin operators. This is summarized in the following self explanatory diagram

$$\begin{array}{c} P_4 \times SO(2)_{\text{Higgs vac}} \\ \updownarrow \\ U(1) \times SU(2) \\ \text{Mixing the Poincaré group with the electroweak symmetry} \end{array}$$

Interestingly, the above indirect mixing of the Poincaré group with the internal symmetries also allows the presence of bosons in the standard model, with spins different from one. In particular, it explains the presence of the spin-0 Higgs itself, and of course of spin-2 fields, which includes the gravitational field.

Indeed, the spin-statistics theorem of quantum mechanics implies that the Lagrangian for a spin-2 field with rest mass  $m$  in the Minkowski space-time, described by a symmetric tensor  $H_{\mu\nu}$ , is given by the Fierz-Pauli Lagrangian in Minkowski space-time (In the following Greek indices run from 1 to 4 and small case Latin indices run from 1 to 3.):

$$\mathcal{L} = \frac{1}{4} [H_{,\mu} H^{,\mu} - H_{\nu\rho,\mu} H^{\nu\rho,\mu} - 2H_{\mu\nu,\mu} H^{,\nu} + 2H_{\nu\rho,\mu} H^{\mu\rho,\nu} - U] \quad (1)$$

where  $\eta_{\mu\nu}$  is the Minkowski metric;  $U = m^2(H_{\mu\nu}H^{\mu\nu} - H^2)$  denotes the potential energy of the field;  $H^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma}H_{\rho\sigma}$  and  $H = \eta^{\mu\nu}H_{\mu\nu} = H_{\mu}^{\mu}$  [6]. The observable degrees of freedom of such field are defined in the vacuum state given by the condition  $\frac{\partial U}{\partial H_{\mu\nu}} = 2m^2(H_{\mu\nu} - H\eta_{\mu\nu}) = 0$ . Assuming  $m \neq 0$ , this vacuum occurs when  $H_{\mu\nu}$  is trace-free  $H = 0$ , which has the effect of reducing the degrees of freedom of the field from six to five. The field equations derived from (1) is the massive wave equation

$$(\Box^2 - m^2)H_{\mu\nu} = 0 \quad (2)$$

It is curious that when  $m = 0$ , this equation resembles the gravitational wave equation as obtained from the linear approximation  $g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$  of Einstein's equations, after applying the de Donder coordinate gauge:

$$\Box^2\psi_{\mu\nu} = 0 \quad \psi_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\gamma\eta_{\mu\nu} \quad (3)$$

This suggests that Einstein's gravitation field is a spin-2 field with zero mass, characterized by the non-linear generalization of (1). Indeed, taking the higher order perturbations of the Minkowski metric by a massless Fierz-Pauli field  $H_{\mu\nu}$ ,  $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + H_{\mu\nu}^2 + \dots$  in the left hand side of (10) for  $m = 0$ , and after a lengthy sequence of term by term comparisons, it was found the same expression of

the Einstein tensor for  $g_{\mu\nu}$ . If we compare this result with an arbitrary conserved energy-momentum tensor  $T_{\mu\nu}^{\text{source}}$ , for all orders of perturbations, then we obtain Einstein's gravitational equations for the field  $g_{\mu\nu}$  in the Minkowski space-time:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu}^{\text{source}}, \quad (4)$$

where  $\kappa$  is just a proportionality constant, not necessarily equal to the Newtonian coupling constant  $8\pi G$  of General Relativity. Regardless if the interpretation of the tensor  $g_{\mu\nu}$ , either as field in Minkowski's space-time or as a manifold metric, the Ricci tensor and scalar have the same formal expressions derived from the Riemann tensor in both cases. Therefore, the equations (4) and the traditional geometric Einstein's equations are both derivable from the same Einstein-Hilbert principle, up to the addition of a divergence-free term [7]

$$\frac{\delta}{\delta g_{\mu\nu}} \int (R - \kappa \mathcal{L}_{\text{source}}) \sqrt{g} = 0 \quad (5)$$

The above non-geometric derivation of Einstein's equations (4) is well known, originally obtained by R. Kraichnan and S. Gupta in the 1950's and discussed on Feynman's Lectures Notes on Gravitation [8–10].

However, the field equations by themselves do not define a theory. To obtain the complete Einstein's gravitational theory (General Relativity), the field  $g_{\mu\nu}$  solution of (4) for  $\kappa = 8\pi G$ , must be interpreted as the metric of a four-dimensional (pseudo) Riemannian manifold, with the Minkowski space-time as its tangent space. In addition, the principles of equivalence and general covariance must be postulated. This implies that the non-linear massless Fierz-Pauli theory defined by (4) and Einstein's gravitational theory have different observers and observables: while in the relativistic non-linear Fierz-Pauli theory represented by (4), the observers are defined by the Poincaré group and the observables are the field components  $H_{\mu\nu}$ , the observers of General Relativity are defined by the diffeomorphism group of the four-dimensional space-times and its observables of the pure gravitational field are defined by the local curvature of the space-time, excluding the gravitational field of the local matter. That is, by the non-trivial solutions of eigenvalue equation for the Weyl tensor  $C_{\mu\nu\rho\sigma}$ :

$$C_{\mu\nu\rho\sigma}X^{\mu\nu} = \lambda X_{\rho\sigma} \quad (6)$$

There are at most six independent real solutions  $\lambda$  of this equation, measured in the 3-dimensional space of the observers [11]. The corresponding eigenvectors  $X^{\mu\nu}$  correspond to the six types I, II, III, N, D and O in the Petrov classification of the Weyl curvature. Type O corresponds to the case where all eigenvalues are zero, so that there are in fact only five non-trivial observables of the gravitational field. Therefore, after implementing the geometric interpretation of  $g_{\mu\nu}$  in (4) as the

metric of a pseudo-Riemannian manifold (defined as the space-time) and after reinstating the principles of general covariance and equivalence, we conclude that Einstein's gravitational field is a massless spin-2 field.

The fact that the geometric and non-geometric derivation of Einstein's equations follow from the same Einstein-Hilbert principle (5), is a strong indicator that the correspondence between Einstein's geometric equation and (4) is unique, up to an additional divergence term. However, such conclusion has been challenged by the supposition that the same non-linear procedure used by Kraichnan and Gupta can also be applied to the Lagrangian (1) with  $m \neq 0$ . The result would be a more general theory of gravitation with mass, from which Einstein's gravitation would be obtained by taking its zero mass limit [12, 13]. However, in 1970 it was found that the mass term in (1) interferes with the Riemann curvature, leading to a non-causal theory, containing ghosts [14]. More recently, it has been shown that the presence of the mass term in Einstein's equations is not compatible with some of the classic experiments of relativistic gravitation [15]. Finally, we may add that there does not make sense to take the zero mass limit in field theory based on the Poincaré mass spectrum. The zero mass limit would require the existence of a *dense set of particles with arbitrarily small masses*. Except for the case of isolated neutrino masses, such dense set is not observable. Therefore, if a massive spin-2 field exists in nature, then it must be defined as being independent of the massless spin-2 field (4).

The differentiable structure of Einstein's equations is globally hyperbolic, means that at any point of the space-time, there exists a 3-dimensional space-like hypersurface  $S$  (a hypersurface for short), orthogonal to a time-like vector field  $\eta^\mu$ , whose integral time-like curve is globally defined. Although  $\eta^\mu$  is not an observable by itself, its projection on  $S$  is an observable, called the extrinsic curvature of  $S$ . Denoting by  $\{e_i^\mu\}$ ,  $i = 1, 2, 3$  a triad of tangent vectors tangent to  $S$ , the extrinsic curvature is the symmetric tensor defined by  $k_{ij} = -2e_{(i}^\mu \eta_{j)}^\nu g_{\mu\nu}$ . It provides a measure of how a hypersurface of the space-time deviates from its tangent plane in each point, independently of the Riemannian curvature of  $S$ . Due to the orthogonality between  $S$  and  $\eta$ , the observers confined to the 3-dimensional hypersurface cannot predict on which direction the 3-dimensional hypersurfaces will turn next, using the Riemannian geometry of  $S$  alone. However, the knowledge of  $k_{ij}$  gives an indication of that direction, by observing the changes of  $k_{ij}$ . In principle such knowledge enables us to set up an initial value problem for the gravitational field [16, 17].

The existence of submanifolds is a classic problem of differential geometry is not straightforward, because the metric geometry of the mother manifold induces the metric of the daughter. Thus the Riemann tensor of former also induces the Riemann tensor of the latter. The re-

lation between these curvatures are the Gauss-Codazzi-Ricci integrability conditions, acting as some sort of constraints, which hold independently of the dynamical equations. For a hypersurface of a curved 4-dimensional space-time these conditions are [18]

$${}^4R_{\mu\nu\rho\sigma}X_{;i}^\mu X_{;j}^\nu X_{;k}^\rho X_{;l}^\sigma = {}^3R_{ijkl} + g^{44}(k_{iK}k_{lJ} - k_{iL}k_{kJ}) \quad (7)$$

$$k_{i[j;k]} = 0,$$

where the indicated covariant derivatives are calculated with  $g_{\mu\nu}$  and  ${}^4R_{ijkl}$  denotes the Riemann tensor of the space-time projected on the hypersurface  $S$ . The Riemann curvature of  $S$  calculated with its own metric  $g_{ij}$  is denoted by  ${}^3R_{ijkl}$ . In these equations  $g_{ij}$  and  $k_{ij}$  are regarded as independent functions of time, but their time dependence appear in the Gauss-Codazzi equations through the Nash flow condition [20]. However, as independent variable they require independent evolution equations.

Using the fact that  $k_{ij}$  is a rank-2 symmetric tensor, its evolution equation may be obtained from the Fierz-Pauli Lagrangian. Defining the particular Fierz-Pauli field

$$H_{\mu\nu} = H_{ij} = k_{ij}, \quad H_{i4} = 0, \quad H_{44} = 0, \quad \forall i, j = 1..3 \quad (8)$$

Its trace  $g^{\mu\nu}H_{\mu\nu} = g^{ij}k_{ij} = h$ , is the mean curvature  $h$  of the hypersurface. The Fierz-Pauli Lagrangian for such field in the curved space-time is

$$\mathcal{L}^k = \frac{1}{4}[h_{,\mu}h^{,\mu} - k_{ik,j}k^{jk,i} - 2k_{ij}{}^{,i}h^{,j} + 2k_{jk;i}k^{ik;j} - U], \quad (9)$$

where now  $U = m^2(k_{ij}k^{ij} - h^2)$ . The field equations is the massive wave equation

$$(\Box^2 - m^2)k_{ij} = 0. \quad (10)$$

and its vacuum state is given by  $\frac{\partial U}{\partial k_{ij}} = 2m^2(k_{ij} - hg_{ij}) = 0$ , so that it occurs when the hypersurface of the space-time is a minimal surface. However, since  $k_{ij}$  is a dynamical field and it interacts (minimally) with the gravitational field, it does not necessarily remain in that vacuum state. It seems natural to assume that the vacuum state occurred in an initial hypersurface, within a Cauchy formulation of the problem of gravitational dynamics.

From the physical point of view, the massive spin-2 field  $k_{ij}$  has the special significance of being a short range carrier of the gravitational interaction, in the sense envisaged by Salam et al, using the analogy with the electric current in electromagnetic theory: the electronic current acts as a short range carrier of the electromagnetic interaction, complementing the long range electromagnetic field itself, and acting as source of Maxwell's equations [19]. The difference is that here we have a tensor current describing the flow of masses from one hypersurface to another [28]. Thus, the evolution of the space-time described by the trajectories of particles belonging to a hypersurface to another produces a deformation of the geometry of the space-time in the sense described by Nash.

The production of mini-black holes at the LHC has been predicted in a higher dimensional space-time, say  $D = 4 + \delta$ , justified by the fact that with large extra dimensions the coupling constant is not necessarily related to the standard Newtonian gravitational constant (4) [23–26]. Here, in the derivation of (4) as a non-linear massless spin-2 field, we have a similar situation where the coupling constant is arbitrary. Thus, that constant may also be adjusted to a wider range of applications in a 4-dimensional space-times, including the production and detection of mini black holes at the current level of proton-proton collisions at the LHC. Thus, taking  $\delta = 0$ , in the expression for the predicted Schwarzschild radius for a mini black hole in four dimensions, we obtain

$$R_{Sch} = \frac{1}{M_{Pl}^2} \frac{\Gamma(3/2)}{\pi^{3/2}} M_{BH} \quad (11)$$

where  $M_{Pl}$  is the Planck mass and  $M_{BH}$  denotes the black hole mass, assumed to be approximately equal to the total energy of the proton-proton collision (currently  $\approx 13TeV$ ).

Summarizing, we have shown that the existence of the Higgs particle inevitably implies in the inclusion of the Poincaré symmetry into the standard model of unification. Such symmetry mixing, in turn implies in the presence of Einstein’s gravitation in that model, as a massless, non-linear spin-2 field. In addition, we also introduce a new spin-2 massive particle of geometrical origin, which can be interpreted as a new, short range carrier of Einstein’s gravitation.

---

[1] L. O’Raifeartaigh, *Mass Differences and Lie Algebras of Finite Order*. Phys. Rev. Lett. **14**, 575 (1965).  
[2] S. Coleman and J. Mandula, *All Possible Symmetries of the S Matrix*. Physical Review **159**, 1251, (1967).  
[3] M. Flato and D. Sternheimer, *Remarks on the Connection Between External and Internal Symmetries*. Phys. Rev. Lett. **15**, 934 (1965).  
[4] M. Flato, C. Fronsdal and D. Sternheimer, *Difficulties with Massless Particles?* Comm. Math. Phys. **90**, 563 (1983).  
[5] A. Pickering, *Constructing Quarks*, U. Chicago Press, Chicago, IL. (1984).  
[6] M. Fierz and W. Pauli, *On Relativistic Wave Equations for Particles of Arbitrary Spin in an Electromagnetic Field*. Proc. Roy. Soc. Lond. A **173**, 211 (1939).  
[7] S. Deser and M. Henneaux, *A Note on Spin Two Fields in Curved Backgrounds*. arXiv:0611157 [gr-qc].  
[8] R.H. Kraichnan, *Special Relativistic Derivation of Generally Covariant Gravitation Theory*. Phys. Rev. **98**, 1118 (1955).  
[9] S. N. Gupta, *Gravitation and Electromagnetism*. Phys. Rev. **96**, 1683, (1954).

[10] R. P. Feynman, *Lectures on Gravitation*. Addison Wesley Publishing Company. Menlo Park, Ca, USA. (1962).  
[11] F. A. E. Pirani, *Invariant Formulation of Gravitational Radiation theory*, Physical Review, **105**, 1089, (1957).  
[12] H. Van Dam and M. J. Veltman, *Massive and Massless Yang-Mills and Gravitational Fields*. Nucl. Phys. B **22**, 397 (1970).  
[13] V. Zakharov, *Linearized Gravitation Theory and the Graviton Mass*. JETP Lett. **12**, 312 (1970).  
[14] D. G. Boulware and S. Deser, *Can Gravitation Have a Finite Range?* Phys. Rev. D **8**, 3368, (1972).  
[15] C. Will, *The Confrontation between General Relativity and Experiment*. ArXiv: 1403.7377V1 [gr-qc].  
[16] Y. Choquet-Bruhat and J. York Jr. *Mathematics of Gravitation*, Vol. I. Banach Center Publication, vol. 41. Institute of Mathematics, Polish Academy of Sciences, Warsaw (1997).  
[17] J. York Jr. *The Initial Value Problem Using Metric and Extrinsic Curvature*, arXiv:gr-qc/0405005v1 (2004).  
[18] L. P. Eisenhart, *Riemannian Geometry*, Princeton University Press, Princeton, N.J. (1966).  
[19] C. J. Isham, A. Salam and J. Strathdee, Phys. Rev. D **3**, 867 (1971).  
[20] J. F. Nash. *The Imbedding Problem for Riemannian Manifolds*. Annals of Maths. **63**, 20 (1956).  
[21] R. S. Hamilton, *Three-manifolds with Positive Ricci Curvature*, J. Diff. Geom. **17**, 255, (1982).  
[22] G. Perelman, *The Entropy Formula for the Ricci Flow and its Geometric Applications*, ArXiv:math/0211159 (2002).  
[23] N. Arkani-Hamed et al, *The Hierarchy problem and new dimensions at a millimeter scale* Phys. Rev. Lett. **B429**, 263 (1998).  
[24] R. C. Myers and M. J. Perry, *Black holes in higher dimensional space-times* Ann. Phys. **172**, 304 (1986).  
[25] Seong Chan Park, *Black Holes at the LHC: A review*. arXiv:1203.4683v1 [hep-ph].  
[26] A. Chamblin, F. Cooper and G.C. Nayak, *Top Quark Production of Black Holes at the CERN LHC*. arXiv:0806.4647v2 [hep-ph].  
[27] Usually referred to as the “spontaneous symmetry breaking of the vacuum symmetry”, the word spontaneous refers to “happening without an apparent external cause” at the expense of the system energy alone. Such spontaneity suggests that the spectrum of eigenvalues of the Casimir mass operator of the Poincaré group, together with the spin spectrum can be considered as a natural number system, even because it results from the observations of the smallest constituents of quantum physics.  
[28] R. Hamilton used Fourier’s heat equation to derive the non-relativistic “Ricci flow”  ${}^3R_{ij} = \frac{1}{2} \frac{\partial g_{ij}}{\partial t}$ , later used to deform hypersurfaces [21, 22]. On the other hand, the solutions of the (7) conditions are usually calculated with the help of analytic expansions. John Nash showed that a differentiable solution of those equations can also be obtained by assuming that in each hypersurface the variables  $k_{ij}$  and  $g_{ij}$  are related as  $k_{ij} = \frac{1}{2} \frac{\partial g_{ij}}{\partial t}$ . This condition is compatible with Einstein’s relativity and it is referred to as the “Nash flow” [20].